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$$\begin{aligned}\cos\varphi\sin(C+\omega) &= \sin\omega[-(x/a)\cos A + (y/b)\cos(\omega-B)] \\ \sin\varphi\sin(C+\omega) &= \sin\omega[(x/a)\sin A - (y/b)\sin(\omega-B)].\end{aligned}$$

Squaring and adding, and transposing the members of the equation, we obtain the required equation of the locus

$$\frac{x^2}{a^2} + \frac{2xy}{ab}\cos(C+\omega) + \frac{y^2}{b^2} = \frac{\sin^2(C+\omega)}{\sin^2\omega}.$$

The locus is therefore an ellipse with center at the intersection of the two lines.

Solved similarly by *G. W. GREENWOOD*, and *G. B. M. ZERR*.

CALCULUS.

163. Proposed by *F. P. MATZ*, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Can there be a plane curve the length of which varies *directly as the abscissa* and *inversely as the ordinate* of any point on the curve?

Solution by *G. B. M. ZERR*, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$S = mx/y, \quad dS = \frac{mydx - mxdy}{y^2} = \sqrt{(dx^2 + dy^2)}.$$

$$\therefore \sqrt{1 + (dx/dy)^2} = [my(dx/dy) - mx]/y^2.$$

Let $x = vy$, $dx/dy = v + y(dx/dy) = v + yp$, suppose.

$$\therefore \sqrt{1 + (v + yp)^2} = mp, \text{ or } v + yp = \sqrt{(m^2 p^2 - 1)} \dots (1).$$

$$\text{From (1), } p = \frac{vy \pm \sqrt{(v^2 m^2 + m^2 - y^2)}}{m^2 - y^2} = \frac{xy \pm \sqrt{[m^2(x^2 + y^2) - y^4]}}{y(m^2 - y^2)}.$$

Differentiating (1) with respect to y ,

$$\frac{dy}{dx} = p = v - y \frac{dp}{dy} + \frac{m^2 p (dp/dy)}{\sqrt{(m^2 p^2 - 1)}}.$$

$$\therefore 2p = \left(\frac{m^2 p}{\sqrt{(m^2 p^2 - 1)}} - y \right) \frac{dp}{dy}.$$

$$\therefore \frac{dy}{dp} = \frac{m^2}{2\sqrt{(m^2 p^2 - 1)}} - \frac{y}{2p} \quad \text{or} \quad \frac{dy}{dp} + \frac{y}{2p} = \frac{m^2}{2\sqrt{(m^2 p^2 - 1)}}.$$

$$\therefore y\sqrt{p} = C + \frac{m^2}{2} \int \frac{\sqrt{p} dp}{\sqrt{(m^2 p^2 - 1)}} = C + \frac{m^2}{2} f(p) \dots (2).$$

The value of p in (2) from (1) gives the primitive, which is the curve desired. By series we get

$$\begin{aligned} \frac{m^2}{2} \int \frac{\nu(p) dp}{\nu(m^2 p^2 - 1)} &= \frac{1}{2} \frac{1}{\nu p} \nu(m^2 p^2 - 1) + \frac{1}{2} m \nu(p) \left(1 + \frac{1}{2.3m^2 p^2} + \frac{1}{2.4.7 m^4 p^4}\right. \\ &\quad \left. + \frac{3}{2.4.6.11 m^6 p^6} + \frac{3.5}{2.4.6.8.15 m^8 p^8} + \dots\right). \end{aligned}$$

164. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $m^2 + n^2 = 1$, $m^2 \cos^2 \theta + n^2 \cos^2 \varphi = A$, $a^2 b^2 \sin^2 \theta (m^2 + n^2 \cos^2 \varphi) + a^2 c^2 \cos^2 \theta \cos^2 \varphi + b^2 c^2 \sin^2 \varphi (n^2 + m^2 \cos^2 \theta) = B$, $\nu(1 - m^2 \sin^2 \theta) = \Delta(\theta)$, $\nu(1 - n^2 \sin^2 \varphi) = \Delta(\varphi)$ prove that $\int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi} \frac{AB d\theta d\varphi}{\Delta(\theta) \Delta(\varphi)} = (\frac{1}{8}\pi)(a^2 b^2 + a^2 c^2 + b^2 c^2)$.

Solution by the PROPOSER.

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$$

$$\int ds = \iint \nu [1 + (dz/dx)^2 + (dz/dy)^2] dx dy = c^2 \iint \frac{\nu [z^2/c^4 + x^2/a^4 + y^2/b^4]}{z} dx dy.$$

Let p = perpendicular from center on tangent plane.

$$\therefore 1/p = \nu [z^2/c^4 + x^2/a^4 + y^2/b^4].$$

$$\text{Let } (b/a)\nu[a^2 - x^2] = y', (a/b)\nu[b^2 - y^2] = x'.$$

$$\begin{aligned} D = \int \frac{ds}{p} &= c^2 \iint \frac{[z^2/c^4 + x^2/a^4 + y^2/b^4]}{z} dx dy = \frac{1}{c^2} \int_0^a \int_0^{y'} z dx dy + \frac{c^2}{a^4} \int_0^a \int_0^{y'} \frac{x^2}{z} dx dy \\ &+ \frac{c^2}{b^4} \int_0^b \int_0^{x'} \frac{(y^2/z)}{z} dy dx = \frac{1}{abc} \int_0^a \int_0^{y'} \nu [a^2 b^2 - b^2 x^2 - a^2 y^2] dx dy \\ &+ \frac{abc}{a^4} \int_0^a \int_0^{y'} \frac{x^2 dx dy}{\nu [a^2 b^2 - b^2 x^2 - a^2 y^2]} + \frac{abc}{b^4} \int_0^b \int_0^{x'} \frac{y^2 dy dx}{\nu [a^2 b^2 - b^2 x^2 - a^2 y^2]} \\ &= \frac{1}{6abc} \pi (ab/c + bc/a + ac/b) = \frac{\pi}{6abc} (a^2 b^2 + b^2 c^2 + a^2 c^2). \end{aligned}$$

$$\text{Let } x = a \sin \varphi \nu/(1 - m^2 \sin^2 \theta), \quad y = b \cos \theta \cos \varphi, \quad z = c \sin \theta \nu/(1 - n^2 \sin^2 \varphi),$$

$$\begin{aligned} m^2 + n^2 &= 1; \quad dx/d\varphi = a \cos \varphi \nu/(1 - m^2 \sin^2 \theta), \quad dx/d\theta = -\frac{am^2 \sin \varphi \sin \theta \cos \theta}{\nu(1 - m^2 \sin^2 \theta)}, \quad dy/d\varphi = \\ &-b \cos \theta \sin \varphi, \quad dy/d\theta = -b \sin \theta \cos \varphi; \quad dx dy = (dx/d\theta)(dy/d\varphi) - (dx/d\varphi)(dy/d\theta) \end{aligned}$$